

Three Parton Corrections in $B \rightarrow PP$ decays

Tsung-Wen Yeh*

*Department of Science Application And Dissemination,
National Taichung University, Taichung 403, Taiwan*

Abstract

The $1/m_b$ corrections from the three parton $q\bar{q}g$ Fock state of the final state light meson in $B \rightarrow PP$ decays are evaluated by means of a collinear expansion method. The impacts of these corrections on the CP averaged branching ratios of the $B \rightarrow \pi K$ decays are analyzed.

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*Electronic address: twyeh@ms3.ntcu.edu.tw

I. INTRODUCTION

The QCD factorization [1, 2, 3] has been widely used to investigate the charmless hadronic B decays. For an operator O_i of the weak effective Hamiltonian, the matrix element for $\bar{B} \rightarrow M_1 M_2$ decays under the QCD factorization is found to be expressible as

$$\begin{aligned} \langle M_1 M_2 | O_i | \bar{B} \rangle = & \sum_j F_j^{B \rightarrow M_1}(m_2^2) \int_0^1 du T_{ij}^I(u) \Phi_{M_2}(u) + (M_1 \leftrightarrow M_2) \\ & + \int_0^1 d\xi du dv T_i^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{M_1}(u) \Phi_{M_2}(v), \end{aligned} \quad (1)$$

where $T^{I(II)}$ denote the parton amplitudes and $\Phi_B, \Phi_{M_1}, \Phi_{M_2}$ represent the light-cone distribution amplitudes (LCDAs) for the initial state \bar{B} meson and the final state M_1 and M_2 mesons, respectively. The parton amplitudes $T^{I(II)}$ contain short distance interactions involved in the decay processes. The LCDAs Φ_B, Φ_{M_1} , and Φ_{M_2} are introduced to account for the long distance interactions. The $F_j^{B \rightarrow M_1}(m_2^2)$ with $j = +, 0$ are the $B \rightarrow M_1$ transition form factors. The meson state vector $|M_i\rangle$, $i = 1, 2$, for the meson M_i is composed of Fock states with different number of partons

$$|M_i\rangle = |q\bar{q}\rangle_{M_i} + |q\bar{q}g\rangle_{M_i} + \cdots. \quad (2)$$

So far, most applications of the factorization formula Eq. (1) are limited to leading Fock state $|q\bar{q}\rangle_{M_i}$ of the light mesons. However, the three parton Fock state $|q\bar{q}g\rangle_{M_i}$ of the M_i meson can also contribute.

The corrections related to the higher Fock state are usually classified as subleading twist contributions, since their contributions are suppressed by factors of $O(1/m_b^n)$ with $n \geq 1$ in comparison with the leading ones. Here, m_b denotes the b quark mass. Within QCD factorization, one can employ the Feynman-diagram approach or the effective-theory approach for studies of subleading twist contributions. There exist established Feynman-diagram approaches for processes other than hadronic decays, such as the calculation scheme for the inclusive hard scattering processes [5, 6, 7] or the method for the exclusive hard scattering processes [29, 30, 31, 32, 33]. However, a systematic Feynman-diagram approach for charmless hadronic B decays is still inaccessible. On the other hand, the effective-theory approaches for charmless hadronic B decays have been extensively investigated in recent years [34, 35, 36, 37, 38, 39].

In this paper, a calculation scheme based on the Feynman-diagram approach for charmless hadronic B decays will be developed. We will concentrate on the construction of this calculation scheme and apply the constructed method to calculate the tree level three parton corrections. The $O(\alpha_s)$ three parton corrections is also desirable to understand their factorization properties. Since the related analysis is tedious, we plan to present the relevant calculations in our another preparing work [44]. The organization of this paper is as following. In Section II, the calculation scheme will be constructed. The factorization of the tree level three parton corrections into the partonic and hadronic parts will be outlined. In Section III, the analysis on how the tree level three parton corrections can be factorized into its partonic and hadronic parts will be described in detail. In Section IV, we will apply the results of the Section III to make predictions for the branching ratios of $B \rightarrow \pi K$ decays. The last section devotes for discussion and conclusion.

II. COLLINEAR EXPANSION AT TREE LEVEL

In this section, we will generalize the collinear expansion method [5, 6, 7] to calculate the three parton corrections from the Fock state $|q\bar{q}g\rangle$ of the meson M_2 in the decay $\bar{B} \rightarrow M_1 M_2$. There exist other types of power corrections, such as the power corrections from soft gluons or renormalons. We identify these as non-partonic power corrections. For these power corrections, our proposed scheme may not be useful. However, to include these non-partonic power corrections requires further assumptions beyond the factorization. For example, the soft gluon power corrections are better determined by nonperturbative theories, such as the QCD sum rules or lattice QCD. In this work, we only investigate how the partonic (or the dynamic) power corrections can be included into the QCD factorization in a consistent way.

The collinear expansion method arises from a motivation of generalizing the leading twist factorization theorem for the hard scattering processes to include the corrections from high Fock states of the target hadrons. The original idea of the collinear expansion method was proposed by Polizer at 1980 [4]. The systematical method was developed by Ellis, Furmanski and Petronzio (EFP) in their pioneer works [5, 6]. Using the collinear expansion, the EFP group showed that, for the DIS processes, the twist-4 power suppressed corrections can be factorized into short distance and long distance parts, which are in a similar factorized form as the leading twist contributions. However, in the EFP's approach, the parton

interpretation for the twist-4 corrections are lost. To recover the parton model picture, Qiu then introduced a Feynman-diagram approach [7] to re-formula the EFP's method. In this Feynman-diagram language, a parton model interpretation for the twist-4 corrections becomes trivial.

To begin with, we first express the matrix element of an operator O_i of the weak effective Hamiltonian H_{eff} of the standard model for the hadronic decays $\bar{B} \rightarrow M_1 M_2$ in terms of parton model amplitudes

$$\begin{aligned}
& \langle M_1 M_2 | O_i | \bar{B} \rangle \\
&= \sum_{j=+,0} F_j^{B \rightarrow M_1}(m_{M_2}^2) \int \frac{d^4 l}{(2\pi)^4} \text{Tr}[T_{ij}^I(l) \Phi_{M_2}(l)] + (M_1 \leftrightarrow M_2) \\
&+ \sum_{j=+,0} F_j^{B \rightarrow M_1}(m_{M_2}^2) \int \frac{d^4 l_1}{(2\pi)^4} \int \frac{d^4 l_2}{(2\pi)^4} \text{Tr}[T_{ij,\mu}^I(l_1, l_2) \Phi_{M_2}^\mu(l_1, l_2)] + (M_1 \leftrightarrow M_2) \\
&+ \int \frac{d^4 l_B}{(2\pi)^4} \frac{d^4 l_{M_1}}{(2\pi)^4} \frac{d^4 l_{M_2}}{(2\pi)^4} \text{Tr}[T_i^{II}(l_B, l_{M_1}, l_{M_2}) \Phi_B(l_B) \Phi_{M_1}(l_{M_1}) \Phi_{M_2}(l_{M_2})], \tag{3}
\end{aligned}$$

where $F_j^{B \rightarrow M_1}$ denote the form factors for $\bar{B} \rightarrow M_1 l \bar{\nu}$ transition. The form factor F_j are defined as

$$\langle M_1(p) | \bar{q} \gamma_\mu b | \bar{B}(P_b) \rangle = F_+(q^2)(p + P_b)_\mu + \frac{F_+(q^2) - F_0(q^2)}{q^2} q_\mu \tag{4}$$

where $q = P_b - p$ and $F_+(q^2) = F_0(q^2)$ under the limit $q^2 \rightarrow 0$. The parton amplitudes $T_{ij}^I(l)$, $T_{ij,\mu}^I(l_1, l_2)$ and $T_i^{II}(l_B, l_{M_1}, l_{M_2})$ are defined to describe the hard scattering center involving four parton, five parton, and six parton interactions corresponding to those diagrams depicted in Fig. 1(a)-(c), respectively. Note that there are also other types of parton amplitudes involving five or six parton interactions not being presented in Fig. 1, which can be attributed to either the physical form factors, or to be of higher twist than three. We have neglected these contributions in Eq. (3). The Tr symbol denotes the trace operation applied on the color and spin indices. For convenience, we employ the light-cone coordinate system such that $P_B^\mu = (p^\mu + q^\mu)$ with two light-like vectors $q^\mu = (q^+, q^-, q_\perp^i) = (Q, 0, 0)$ and $p^\mu = (p^+, p^-, p_\perp^i) = (0, Q, 0)$ with $Q = m_B/\sqrt{2}$, which are defined as the momenta carried by the final state M_2 and M_1 mesons, respectively. The M_1 meson is defined to receive the spectator quark of the bottom meson. The M_2 meson is defined as the emitted meson produced from the hard scattering center. The hadron amplitudes Φ_{M_2} and $\Phi_{M_2}^\mu(l_1, l_2)$ are

defined as

$$\Phi_{M_2}(l) = \int d^4y e^{il \cdot y} \langle M_2 | \bar{q}(y) q(0) | 0 \rangle , \quad (5)$$

$$\Phi_{M_2}^\mu(l_1, l_2) = \int d^4y \int d^4z e^{il_1 \cdot y} e^{i(l_2 - l_1) \cdot z} \langle M_2 | \bar{q}(y) (-g A^\mu(z)) q(0) | 0 \rangle . \quad (6)$$

In our language, Eq. (3) contain leading, sub-leading and higher twist contributions. The Eq. (3) becomes meaningful only if the leading twist contributions can be separated from the sub-leading twist contributions. For this purpose, we employ the Qiu's Feynman-diagram collinear expansion approach [7] to expand each Feynman diagram in a twist by twist manner. As the loop corrections are considered, the twist expansion then interplays with the expansion in α_s . To be specific, we choose the following expansion strategy.

1. All possible Feynman diagrams ordered in α_s are first drawn.
2. According to the collinear expansion (developed below), each Feynman diagram of order $O(\alpha_s^n)$ with $n \geq 0$ is expanded into a series ordered by increasing twist.
3. The contributions of the same twist order from the expansion series of each Feynman diagram with the same α_s order are added up together.
4. The factorization properties of the final expression with a specific twist and a specific α_s order are analyzed.

The last one is important for us to derive a meaningful perturbation theory beyond the leading twist.

For latter uses, we define the soft, collinear and hard loop parton momenta. We let the soft momentum scale as $(l^+, l^-, l_\perp) \sim (\lambda, \lambda, \lambda)$, the collinear momentum scale as $(l^+, l^-, l_\perp) \sim (Q, \lambda^2/Q, \lambda)$, and the hard momentum scale as $(l^+, l^-, l_\perp) \sim (Q, Q, Q)$. The scale variables are defined as $Q \sim m_b$ and $\lambda \sim \Lambda_{QCD}$. For a collinear loop parton, it is convenient to parametrize its momentum l^μ into its components proportional to the meson momentum q^μ , the light-cone vector n^ν , and the transversal directions

$$l^\mu = n \cdot l q^\mu + \frac{l^2 + l_\perp^2}{2n \cdot l} n^\mu + l_\perp^\mu , \quad (7)$$

where the vector n^μ satisfies $n \cdot q = 1$, $n \cdot l_\perp = 0$, and $n^2 = 0$. For convenience, we further define the collinear component, \hat{l}^μ , the on-shell component, l_L^μ , and the off-shell component,

l_S^μ of the momentum l^μ as

$$\begin{aligned}\hat{l}^\mu &= n \cdot l q^\mu , \\ l_L^\mu &= \hat{l}^\mu + \frac{l_\perp^2}{2n \cdot l} n^\mu + l_\perp^\mu , \\ l_S^\mu &= \frac{l^2}{2n \cdot l} n^\mu .\end{aligned}\tag{8}$$

In the above expansion of the parton momentum into different parts, we have assumed $m_{M_i} = 0$, $i = 1, 2$, and $q^2 = 0$. The loop partons except of the bottom quark are assumed massless for simplicity. The contributions from non-vanishing light quark masses are taken as corrections. Because the light quark mass contributions are relatively negligible as compared to the bottom quark mass, we also neglect the light quark mass effects in the following calculations.

According to the parametrization in Eq. (7), a parton propagator can be separated into its long distance part and short distance part (the special propagator). If we write the loop parton propagator as [7]

$$\begin{aligned}F(y, z) &= \int \frac{d^4 l}{(2\pi)^4} e^{i l \cdot (y - z)} [F_L(l) + F_S(l)] \\ &= F_L(y, z) + F_S(y, z) ,\end{aligned}\tag{9}$$

where

$$F_L(l) = \frac{i \not{l}_L}{l^2} , \quad F_S(l) = \frac{i \not{l}}{2n \cdot l} .\tag{10}$$

The $F_L(l)$ propagator corresponds to the long distance part of the propagator, since $F_L(y, z) \propto \theta(y - z)$. The $F_S(l)$ propagator represents the short distance part because $F_S(y, z) \propto \delta(y - z)$. We now describe one important property of the long distance propagator $F_L(l)$. In a parton amplitude, the $F_L(l)$ may contact with a $\not{q} n^\mu$ component of a vertex γ^μ . Whenever this happens, the $\not{q} n^\mu$ vertex will extract one short distance propagator $F_S(l)$ and one interaction vertex $i\gamma_\nu$ from the relevant hadron amplitude [7]

$$\frac{i \not{l}_L}{l^2} \not{q} = \frac{i \not{l}_L}{l^2} (i\gamma_\nu) \frac{i \not{l}}{2n \cdot l} \not{q} (l - \hat{l})^\nu .\tag{11}$$

The momentum factor $(l - \hat{l})^\nu$ is then absorbed by the hadron amplitude due to the Ward identity [7]. We now explain how the identity Eq. (11) can be obtained by a simple manipulation. We first insert an identity $1 = (\not{l}^2)/l^2$ into the left hand side of Eq. (11) and expresse

each \not{l} into $\not{l}_L + \not{l}_S$ to obtain

$$\frac{i\not{l}_L}{l^2} \frac{\not{l}}{l^2} \not{q} = \frac{i\not{l}_L}{l^2} \frac{(\not{l}_L + \not{l}_S)(\not{l}_L + \not{l}_S)}{l^2} \not{q}. \quad (12)$$

Since $(\not{l}_L)^2 = 0 = (\not{l}_S)^2$, the above equation then becomes

$$\frac{i\not{l}_L}{l^2} \frac{\not{l}}{l^2} \not{q} = \frac{i\not{l}_L}{l^2} \frac{(\not{l}_L \not{l}_S + \not{l}_S \not{l}_L)}{l^2} \not{q}, \quad (13)$$

where the first term $\not{l}_L \not{l}_S$ in the right hand side leads to a vanishing result as it contacts with the term $i\not{l}_L/l^2$ term. The only contribution can only come from the second term $\not{l}_S \not{l}_L$ in the right hand side of Eq. (13). In addition, the \not{l}_L can be expanded in the terms proportional to \not{q} , \not{p} , \not{l}_\perp . This gives

$$\not{l}_S \not{l}_L \not{q} = l^2 \frac{\not{p}}{2n \cdot l} (n \cdot l \not{q} + \frac{l_\perp^2 \not{p}}{2n \cdot l} + \not{l}_\perp) \not{q}.$$

Due to $\not{q}^2 = \not{p}^2 = 0$, it further reduces to

$$l^2 \frac{\not{p}}{2n \cdot l} (\not{l}_\perp) \not{q}.$$

By substituting the above back into Eq. (12), Eq. (11) is then obtained by noting that

$$\not{l}_L (i\gamma_\alpha) (i\not{p}) \not{q} (l - \hat{l})^\alpha = \not{l}_L \not{p} \not{l}_\perp \not{q}.$$

Using Eq. (11), one can systematically include the effects from the non-collinearity and the off-shellness of the collinear partons. This property of the long distance part of the parton propagator plays an important role in our following analysis, and its meaning will become more clear after we have investigated real cases lately.

According to the parton model, the hadron amplitudes are defined as the probability for finding the on-shell partons inside the hadron. The parton amplitudes are then required to contain only the on-shell components of the external parton momenta. However, according to Eq. (8), either the on-shell momentum l_L or the collinear momentum \hat{l} can be assigned for an on-shell parton. Therefore, there arise two factorization schemes, the collinear factorization [8, 9, 10, 11, 12, 13] (QCD factorization) and the k_T factorization [14, 15, 16, 17] (PQCD factorization [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]). In the k_T factorization scheme, an on-shell parton carries a momentum l_L . On the other hand, in the collinear factorization scheme, an on-shell parton carries a momentum \hat{l} . In this work, we follow the QCD factorization to use the collinear factorization scheme as our basics. Our proposed collinear expansion method is composed of following steps:

1. Use scale analysis for the parton amplitudes according to the scales of parton momenta to find out the leading regions of the parton momentum configuration.
2. The parton amplitudes are expanded into a Taylor series with respect to the leading regions of parton momenta.
3. The expanded parton amplitudes are substituted back into the contraction with the hadron amplitudes to extract relevant contributions up to specific twist order.
4. The factorization of parton momentum integrals is accomplished by means of integral transformations (See, for example, Eq. (24)).
5. The color structure of the parton amplitude is extracted to be attributed to the hadron amplitudes to complete the color factorization.
6. The factorization of spin indices is completed by means of Fierz transformation.
7. The property of the long distance parton propagator is used to extract higher twist contributions.

We are now ready to discuss the collinear expansion. First, we order the parton amplitudes in α_s

$$T_{ij}^I(l) = T_{ij}^{I(0)} + T_{ij}^{I(1)}(l) + O(\alpha_s^2) , \quad (14)$$

$$T_{ij,\mu}^I(l_1, l_2) = T_{ij,\mu}^{I(0)}(l_1, l_2) + T_{ij,\mu}^{I(1)}(l_1, l_2) + O(\alpha_s^2) , \quad (15)$$

$$T_i^{II}(l_B, l_{M_1}, l_{M_2}) = T_i^{II(1)}(l_B, l_{M_1}, l_{M_2}) + O(\alpha_s^2) , \quad (16)$$

where the superscription (0) and (1) are used to denote the relevant parton amplitude of zeroth and first order in α_s , respectively. The parton amplitude $T_{ij}^{I(0)}$ is just the tree vertex $\bar{\Gamma}_i \delta_{ij}$ in the diagram as depicted in Fig. 2. There are vertex and penguin diagrams for $T_{ij}^{I(1)}$ amplitudes as depicted in Fig. 4. The parton amplitude $T_{ij,\mu}^{I(0)}$ describes the tree diagrams as depicted in Fig. 3, in which two quark partons and one gluon parton from the meson M_2 are interacting with a local four fermion operator $O_i = (\bar{q}_1 \Gamma_i b)(\bar{q}_2 \bar{\Gamma}_i q_3)$ with $\Gamma_i (\bar{\Gamma}_i)$ the Dirac gamma matrix for the operator. The parton amplitude T_i^{II} starts from $O(\alpha_s)$ diagrams as depicted in Fig. 5 and is denoted as $T^{II(1)}$.

We first expand the parton amplitudes $T_{ij,\mu}^{I(0)}$ with respect to the collinear components of their relevant parton momenta as

$$T_{ij,\mu}^{I(0)}(l_1, l_2) = T_{ij,\mu}^{I(0)}(\hat{l}_1, \hat{l}_2) + \sum_{k=1}^2 \left. \frac{\partial T_{ij,\mu}^{I(0)}}{\partial l_k^\nu} \right|_{l_k=\hat{l}_k} (l_k - \hat{l}_k)^\nu + \cdots, \quad (17)$$

where

$$T_{ij,\mu}^{I(0)}(\hat{l}_1, \hat{l}_2) = ((i\gamma_\mu) \frac{i\hat{l}_2}{\hat{l}_2^2} \bar{\Gamma}_i + \bar{\Gamma}_i \frac{-i\hat{l}_1}{\hat{l}_1^2} (-i\gamma_\mu)) \delta_{ij}, \quad (18)$$

$$\frac{\partial T_{ij,\mu}^{I(0)}}{\partial l_k^\nu}(\hat{l}_1, \hat{l}_2) = ((i\gamma_\mu) \frac{i\hat{l}_2}{\hat{l}_2^2} (i\gamma_\nu) \frac{i\hat{l}_2}{\hat{l}_2^2} \bar{\Gamma}_i \delta_{k2} - \bar{\Gamma}_i \frac{-i\hat{l}_1}{\hat{l}_1^2} (-i\gamma_\nu) \frac{-i\hat{l}_1}{\hat{l}_1^2} (-i\gamma_\mu) \delta_{k1}) \delta_{ij}. \quad (19)$$

The expansion series are then substituted back into the convolution integrals with the hadron amplitudes for further analysis. The reason why one can expand the parton amplitudes with respect to the relevant collinear momenta will be explained in detail below.

A. Expansion with $T_{ij}^{I(0)}$

The expression for the contributions associated with $T_{ij}^{I(0)}$ is written as

$$\sum_{j=+,0} F_j^{B \rightarrow M_1}(m_{M_2}^2) \int \frac{d^4 l}{(2\pi)^4} \text{Tr}[T_{ij}^{I(0)} \Phi_{M_2}(l)], \quad (20)$$

where the loop momentum l is carried by the loop parton in Fig. 1(a). Since the parton amplitude $T_{ij}^{I(0)}$ is independent of l , we propose to use the following integral identity to transform the expression into a form consistent with the parton model picture

$$\begin{aligned} & \int_0^1 dx \delta(x - n \cdot l) \\ &= \int_0^1 dx \int_0^\infty \frac{d\lambda}{2\pi} e^{i\lambda(x - n \cdot l)} \\ &= 1. \end{aligned} \quad (21)$$

The transformed result appears as

$$\int_0^1 dx \text{Tr}[T_{ij}^{I(0)} \Phi_{M_2}(x)] \quad (22)$$

where

$$\Phi_{M_2}(x) = \int_0^\infty \frac{d\lambda}{2\pi} e^{i\lambda x} \langle M_2 | \bar{q}(\lambda n) q(0) | 0 \rangle. \quad (23)$$

The following integral transformation has been used in the above to simplify the expression

$$\begin{aligned}
& \int \frac{d^4 l}{(2\pi)^4} \int d^4 y e^{il \cdot (y - \lambda n)} G(y, 0) \\
&= \int d^4 y \delta^{(4)}(y - \lambda n) G(y, 0) \\
&= G(\lambda n, 0) ,
\end{aligned} \tag{24}$$

where $G(y, 0)$ denotes any function of the coordinates. Two comments for the above integral transformations Eqs. (21) and (24) are necessary. First, the momentum fraction x for the parton of the meson M_2 is introduced. Second, the quark field $\bar{q}(\lambda n)$ is ordered in light-cone direction n . This implies that the hadron amplitude $\Phi_{M_2}(x)$ is defined on the light-cone $n^2 = 0$ where n^μ is a null light-cone vector. By using the above integral transformations, the parton amplitude and the hadron amplitude are only related by the momentum fraction x . Because the parton amplitude $T_{ij}^{I(0)}$ is equal to $\bar{\Gamma}_i \delta_{ij}$, the integral over x is then associated with $\Phi_{M_2}(x)$.

The factorization of spin indices depends on the structure of $\bar{\Gamma}_i$. For $(V - A)(V \pm A)$ operators, $\bar{\Gamma}_i = (V \pm A)$ and can be expanded into $\not{d}(1 \pm \gamma_5)$, $\not{n}(1 \pm \gamma_5)$ and $\gamma_\perp \gamma_5$. If M_2 is a pseudo-scalar meson, only the axial vector part can contribute. However, only $\not{n} \gamma_5$ leads to leading twist contributions. The $\not{d} \gamma_5$ will result in twist-4 contributions and $\gamma_\perp \gamma_5$ will not contribute. For other types of meson, similar considerations can be made. We now explain how the $\not{d} \gamma_5$ part can contribute. The long distance part of the parton propagator can interact with vertex $\not{d} \gamma_5$ to have

$$\frac{i \not{L}}{l^2} \not{d} = \frac{i \not{L}}{l^2} (i \gamma_\alpha) \frac{i \not{n}}{2n \cdot l} \not{d} (l - \hat{l})^\alpha . \tag{25}$$

It is also applicable for the other parton propagator of the anti-quark line. The short distance part of the parton propagator $i \not{n} / (2n \cdot l)$ and the vertex $i \gamma_\alpha$ are absorbed into the parton amplitude. This results in

$$\int_0^1 dx \text{Tr}[T_{ij, \alpha\beta}(x, x, x) w_\alpha^\alpha w_{\beta'}^\beta \Phi_{M_2, \partial}^{\alpha' \beta'}(x, x, x)] \tag{26}$$

where $w_{\alpha'}^\alpha = g_{\alpha'}^\alpha - q^\alpha n_{\alpha'}$,

$$\begin{aligned}
T_{ij, \alpha\beta}(x, x, x) &\equiv (i \gamma_\alpha) \frac{i \not{n}}{2n \cdot l} T_{ij}^{I(0)} \frac{-i \not{n}}{2n \cdot \bar{l}} (-i \gamma_\beta) , \\
\Phi_{M_2, \partial}^{\alpha' \beta'}(x, x, x) &= \int_0^\infty \frac{d\lambda}{2\pi} e^{i\lambda x} \langle M_2 | \bar{q}(\lambda n) i \partial^{\alpha'}(\lambda n) i \partial^{\beta'}(\lambda n) q(0) | 0 \rangle .
\end{aligned} \tag{27}$$

There are corresponding contributions from the two gluon insertion diagrams depicted in Fig. 6, whose expression is written as

$$\int \frac{d^4 l_1}{(2\pi)^4} \frac{d^4 l_2}{(2\pi)^4} \frac{d^4 l_3}{(2\pi)^4} \text{Tr}[T_{ij,\alpha\beta}^{I(0)}(l_1, l_2, l_3) w_{\alpha'}^\alpha w_{\beta'}^\beta \Phi_{M_2,A}^{\alpha\beta}(l_1, l_2, l_3)] \quad (28)$$

where we have employed the light-cone gauge $n \cdot A = 0$ for the gluon fields, and the parton amplitude and hadron amplitude are expressed as

$$\begin{aligned} T_{ij,\alpha\beta}^{I(0)}(l_1, l_2, l_3) &= (i\gamma_\alpha) \frac{i\not{l}}{2n \cdot l_2} T^{I(0)} \frac{-i\not{l}}{2n \cdot l_2} (-i\gamma_\beta) \\ \Phi_{M_2,A}^{\alpha\beta}(l_1, l_2, l_3) &= \int d^4 z \int d^4 y \int d^4 w e^{il_1 \cdot y} e^{i(l_2 - l_1) \cdot z} e^{i(l_3 - l_2) \cdot w} \\ &\times \langle M_2 | \bar{q}(y) (-gA^\alpha(z)) (-gA^\beta(w)) q(0) | 0 \rangle \end{aligned} \quad (29)$$

Since $T_{ij,\alpha\beta}^{I(0)}(l_1, l_2, l_3)$ can be replaced by $T_{ij,\alpha\beta}^{I(0)}(x_1, x_2, x_3)$ straightforwardly, the momentum integrations over l_1, l_2, l_3 can be transformed into the integrations over x_1, x_2, x_3 . We then obtain

$$\int dx_1 dx_2 dx_3 \text{Tr}[T_{ij,\alpha\beta}^{I(0)}(x_1, x_2, x_3) \Phi_{M_2,A}^{\alpha\beta}(x_1, x_2, x_3)] . \quad (30)$$

The combination of Eq. (26) and Eq. (30) gives

$$\int dx_1 dx_2 dx_3 \text{Tr}[T_{ij,\alpha\beta}^{I(0)}(x_1, x_2, x_3) w_{\alpha'}^\alpha w_{\beta'}^\beta \Phi_{M_2,D}^{\alpha'\beta'}(x_1, x_2, x_3)] \quad (31)$$

where

$$\begin{aligned} \Phi_{M_2,D}^{\alpha\beta}(x_1, x_2, x_3) &= \int \frac{d\lambda}{2\pi} \int \frac{d\eta}{2\pi} \int \frac{d\omega}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2 - x_1)} e^{i\omega(x_3 - x_2)} \\ &\times \langle M_2 | \bar{q}(\lambda n) (iD^\alpha(\eta n)) (iD^\beta(\omega n)) q(0) | 0 \rangle \end{aligned} \quad (32)$$

with $iD^\alpha = i\partial^\alpha - gA^\alpha$ being the covariant derivative. Since \not{n} is of $O(Q^{-1})$, $T_{ij,\alpha\beta}^{I(0)}$ is of $O(Q^{-2})$ as the scale of $T_{ij}^{I(0)}$ being of $O(1)$. The relevant contributions are of higher than twist-4. The above example is to show that, using the collinear expansion, one can calculate the tree level higher twist corrections from the dynamical partons in a systematic way. Because we only intend to calculate the twist-3 corrections, we will not further explore the contributions of twist order higher than three. For $-2(S - P)(S + P)$ operators, $\bar{\Gamma}_i = \gamma_5$. Up to twist-3, the expression appears as

$$\begin{aligned} \int_0^1 dx \text{Tr}[T_{ij}^{I(0)} \Phi_{M_2}(x)] &= -\frac{1}{4} \int_0^1 dx \text{Tr}[T_{ij}^{I(0)} \not{x} \gamma_5] \text{Tr}[\not{x} \gamma_5 \Phi_{M_2}(x)] \\ &+ \frac{1}{4} \int_0^1 dx \text{Tr}[T_{ij}^{I(0)} \gamma_5] \text{Tr}[\gamma_5 \Phi_{M_2}(x)] . \end{aligned} \quad (33)$$

By identifying $\text{Tr}[\not{n}\gamma_5\Phi_{M_2}(x)]$ and $\text{Tr}[\gamma_5\Phi_{M_2}(x)]$ as the twist-2 and twist-3 two parton LCDAs of the M_2 meson

$$\text{Tr}[\not{n}\gamma_5\Phi_{M_2}(x)] = -if_{M_2}\phi_{M_2}^{tw2}(x), \quad (34)$$

$$\text{Tr}[\gamma_5\Phi_{M_2}(x)] = -if_{M_2}\mu_\chi\phi_{M_2,P}^{tw3}(x), \quad (35)$$

where $\mu_\chi^{M_2} = m_{M_2}^2/(\bar{m}_q + \bar{m}_{\bar{q}})$ with \bar{m}_q and $\bar{m}_{\bar{q}}$ the current quark masses and m_{M_2} the meson mass, we recover the naive factorization result up to twist-3 order.

B. Expansion with $T_{ij,\mu}^{I(0)}$

We show the light-cone gauge $n \cdot A = 0$ and the covariant gauge $\partial \cdot A = 0$ for the expansion with $T_{ij,\mu}^{I(0)}$. Because the analysis is tedious, we outline the procedure, here, and leave the details for next section. The first step is to take a power counting for the parton amplitude $T_{ij,\mu}^{I(0)}$. There are three interesting regions. The first region is composed of either two soft loop parton momenta, or one soft loop parton momentum and one collinear loop parton momentum. The $T_{ij,\mu}^{I(0)}$ in the first region is counted as λ^{-1} . The second region is composed of two collinear loop parton momenta. The $T_{ij,\mu}^{I(0)}$ in the second region is counted as $Q\lambda^{-2}$. The third region is composed of either one collinear loop parton momentum and one hard loop parton momentum, or two hard loop parton momenta. In the third region, the $T_{ij,\mu}^{I(0)}$ is counted as Q^{-1} . We conclude that the region composed of two collinear loop parton momenta is dominant.

Let's, first, consider the light-cone gauge $n \cdot A = 0$. The expansion series of $T_{ij,\mu}^{I(0)}(l_1, l_2)$ with respect to \hat{l}_i , $i = 1, 2$, are written as

$$T_{ij,\mu}^{I(0)}(l_1, l_2) = T_{ij,\mu}^{I(0)}(\hat{l}_1, \hat{l}_2) + \sum_{k=1,2} T_{ijk,\mu\nu}^{I(0)}(\hat{l}_1, \hat{l}_k, \hat{l}_2)(l_k - \hat{l}_k)^\nu + \dots, \quad (36)$$

where $T_{ijk,\mu\nu}^{I(0)}(\hat{l}_1, \hat{l}_k, \hat{l}_2)$ are defined by assuming the low energy theories

$$T_{ijk,\mu\nu}^{I(0)}(\hat{l}_1, \hat{l}_k, \hat{l}_2) = \left. \frac{\partial T_{ij,\mu}^{I(0)}}{\partial l_k^\nu} \right|_{l_1=\hat{l}_1, l_2=\hat{l}_2}. \quad (37)$$

The expansion series are then substituted back into the convolution integrals. Since the gauge condition $n \cdot A = 0$ with a light-cone vector n^μ satisfying $n \cdot q = 1$, $n^2 = 0$, and $n \cdot l_\perp = 0$, the first term $T_{ij,\mu}^{I(0)}(\hat{l}_1, \hat{l}_2)$ leads to the result

$$\int \frac{d^4 l_1}{(2\pi)^4} \int \frac{d^4 l_2}{(2\pi)^4} \text{Tr}[T_{ij,\mu}^{I(0)}(\hat{l}_1, \hat{l}_2) w_\alpha^\mu \Phi_{M_2}^\alpha(l_1, l_2)] \quad (38)$$

where we have introduced $w_\alpha^\mu = g_\alpha^\mu - q^\mu n_\alpha$. Similarly, we employ the following integral identities to simplify the momentum integrations

$$\int_0^1 dx_1 \int_0^1 dx_2 \int_0^\infty \frac{d\lambda}{2\pi} \int_0^\infty \frac{d\eta}{2\pi} e^{i\lambda(x_1 - n \cdot l_1)} e^{i\eta(x_2 - n \cdot l_2)} = 1 \quad (39)$$

The expression appears as

$$\int_0^1 dx_1 \int_0^1 dx_2 \text{Tr}[T_{ij,\mu}^{I(0)}(x_1, x_2) w_\alpha^\mu \Phi_{M_2}^\alpha(x_1, x_2)] \quad (40)$$

in which

$$T_{ij,\mu}^{I(0)}(x_1, x_2) = T_{ij,\mu}^{I(0)}(\hat{l}_1, \hat{l}_2) \Big|_{\hat{l}_1=x_1, \hat{l}_2=x_2}, \quad (41)$$

$$\begin{aligned} \Phi_{M_2}^\alpha(x_1, x_2) &= \int_0^\infty \frac{d\lambda}{2\pi} \int_0^\infty \frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2 - x_1)} \\ &\times \langle M_2 | \bar{q}(\lambda n) (-g A^\mu(\eta n)) q(0) | 0 \rangle. \end{aligned} \quad (42)$$

In the above equations, we have used the following transformation for any function $G(y, z, 0)$ of coordinates y^μ and z^μ

$$\begin{aligned} &\int \frac{d^4 l_1}{(2\pi)^4} \int \frac{d^4 l_2}{(2\pi)^4} \int d^4 y \int d^4 z e^{il_1 \cdot (y - z - \lambda n)} e^{il_2 \cdot (z - \eta n)} G(y, z, 0) \\ &= G(\lambda n, \eta n, 0). \end{aligned} \quad (43)$$

For tree diagrams, only color singlet operators can contribute at twist-3. The remaining task is to finish the factorization of spin indices. For $(V - A)(V \pm A)$ operators, the related contributions are of twist-4, which are beyond our accuracy. For $-2(S - P)(S + P)$ operators, the related contributions are of twist-3. To obtain the collinear limit $\hat{l}_i \rightarrow x_i$, $i = 1, 2$, for $T_{ij,\mu}^{I(0)}(x_1, x_2)$, we have used the following substitution for the parton propagators

$$\frac{i \hat{\not{l}}_1}{\hat{l}_1^2} \rightarrow \frac{i \not{n}}{2n \cdot l_1}. \quad (44)$$

This is because the vertex $i\gamma_\mu$ in $T_{ij,\mu}^{I(0)}(x_1, x_2)$ is transversal and only the off-shell part of the parton propagators can contribute. See further explanations in the next section. The terms associated with $T_{ijk,\mu\nu}^{I(0)}$ are of twist-4 and higher. They are neglected accordingly. The factorization of spin indices results in

$$\frac{1}{8} \int_0^1 dx_1 \int_0^{\bar{x}_1} dx_2 \text{Tr}[T_{ij,\mu}^{I(0)} \sigma_{\alpha\beta} \gamma_5] w_{\mu'}^\mu \text{Tr}[\sigma^{\alpha\beta} \gamma_5 \Phi_{M_2}^{\mu'}(x_1, x_2)]. \quad (45)$$

The explicit expression for Eq. (45) is left to the next section.

We now consider the expansion with covariant gauge $\partial \cdot A = 0$. Since the factorizations of the momentum integrals and color indices are independent of gauge condition, we can go through to consider the factorization of spin indices. The first term in the expansion of $T_{ij,\mu}^{I(0)}(x_1, x_2)$, under covariant gauge, is related to the gauge invariant phase factor of the related two parton amplitudes. The gluon fields A^μ in $\Phi_{M_2}^\mu$ can be expanded as $A^\mu = n \cdot A q^\mu + q \cdot A n^\mu + d_\alpha^\mu A^\alpha$. The contraction $\text{Tr}[T_{ij,\mu}^{I(0)}(x_1, x_2) q^\mu n \cdot \Phi_{M_2}(x_1, x_2)]$ leads to

$$\int dx_1 \int dx_2 \text{Tr}[\bar{\Gamma}_i \frac{2}{n \cdot k} n \cdot \Phi_{M_2}(x_1, x_2)] , \quad (46)$$

where $k = l_2 - l_1$ being the gluon momentum and

$$n \cdot \Phi_{M_2}(x_1, x_2) = \int_0^\infty \frac{d\lambda}{2\pi} \int_0^\infty \frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2 - x_1)} \langle M_2 | \bar{q}(\lambda n) (-g n \cdot A(\eta n)) q(0) | 0 \rangle . \quad (47)$$

The terms with $q \cdot A n^\mu$ vanish since the covariant gauge condition $\partial \cdot A = 0$. The terms with the contraction $\text{Tr}[T_{ij,\mu}^{I(0)}(x_1, x_2) d_\alpha^\mu \Phi_{M_2}^\alpha(x_1, x_2)]$ are of higher twist than twist-3 and will be neglected. With the above considerations, the contraction $\text{Tr}[T_{ij,\mu}^{I(0)}(x_1, x_2) \Phi_{M_2}^\mu(x_1, x_2)]$ leads to contributions of twist-2 or higher than twist-3.

We next consider the contraction $\text{Tr}[T_{ijk,\mu\nu}^{I(0)}(x_1, x_2) (l_k - \hat{l}_k)^\nu \Phi_{M_2}^\mu(x_1, x_2)]$, which can be rewritten as

$$\text{Tr}[T_{ijk,\mu\nu}^{I(0)}(x_1, x_k, x_2) w_\nu^\nu \Phi_{M_2}^{\nu'\mu}(x_1, x_k, x_2)] \quad (48)$$

with

$$\Phi_{M_2}^{\nu'\mu}(x_1, x_k, x_2) \equiv \int \frac{d\lambda}{2\pi} \int \frac{d\eta}{2\pi} e^{i\lambda x_1} e^{i\eta(x_2 - x_1)} \langle M_2 | \bar{q}_2(\lambda n) i g G^{\nu'\mu}(\eta n) q_3(0) | 0 \rangle . \quad (49)$$

Note that only transversal part $d_{\perp,\beta}^\nu (l_k - \hat{l}_k)^\beta$ of the $(l_k - \hat{l}_k)^\nu$ can contribute at twist-3. For $(V - A)(V \pm A)$ operators, the contributions are of twist-4. For $-2(S - P)(S + P)$ operators, the result appears as

$$\begin{aligned} & \frac{1}{8} \int_0^1 dx_1 \int_0^{\bar{x}_1} dx_2 \int dx_k \text{Tr}[T_{ijk,\mu\nu}^{I(0)}(x_1, x_k, x_2) d_{\perp,\nu'}^\nu \sigma_{\alpha\beta} \gamma_5] [\sigma^{\alpha\beta} \gamma_5 \Phi_{M_2}^{\nu'\mu}(x_1, x_k, x_2)] \\ & \times (\delta(x_k - x_1) + \delta(x_k - x_2)) . \end{aligned} \quad (50)$$

The reader may have noticed that the terms in the expansion series of $T_{ij,\mu}^{I(0)}(l_1, l_2)$ in Eq. (36) are of different twist order under the covariant or the light-cone gauge. For example, the twist-3 contributions are from the the first term in the expansion series of $T_{ij,\mu}^{I(0)}(l_1, l_2)$ under

the light-cone gauge. On the other hand, under the covariant gauge, the twist-3 contributions are from the second term in the expansion series of $T_{ij,\mu}^{I(0)}(l_1, l_2)$. Since the parton amplitude and hadron amplitude under the collinear expansion are required to be gauge invariant, respectively, this feature of the collinear expansion method can be used as a guiding principle for calculations.

III. TWIST-3 CORRECTIONS

In this section, we make a more detail descriptions for the twist-3 contributions from the three parton Fock state $q\bar{q}g$ of the M_2 meson. The amplitude for the three parton $q\bar{q}g$ of M_2 interacting with the operator O_6 at the tree level for $\bar{B} \rightarrow M_1 M_2$ decays is written as

$$\begin{aligned} & \langle M_1 | \bar{q}_1(0)(1 - \gamma_5)b(0) | \bar{B} \rangle \\ & \times \int d^4y \int d^4z \int \frac{d^4l}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} e^{il \cdot z} e^{ik \cdot (y-z)} \langle M_2 | \bar{q}_2(z) [(-ig\not{A}(y)) \frac{i(\not{y} + \not{k})}{(l+k)^2 + i\epsilon} (1 + \gamma_5) \\ & + \frac{-i(\not{q} - \not{l} + \not{k})}{(q-l+k)^2 + i\epsilon} (+ig\not{A}(y))(1 + \gamma_5)] q_3(0) | 0 \rangle . \end{aligned} \quad (51)$$

The l and k denote the momenta carried by the q_2 quark and g gluon fields in Fig. 3(a) and (b). We first employ the light-cone gauge $n \cdot A(y) = 0$. The gluonic fields $A^\alpha(y)$ represents $A^{\alpha,a}(y)T^a$ with the color matrix T^a in the fundamental representation $\sum T^a T^b = \delta^{ab}/2$. To relate to the previous introduced collinear expansion, we recast the convolution integration part of Eq. (51) into the form

$$\int \frac{d^4l}{(2\pi)^4} \frac{d^4k}{(2\pi)^4} \text{Tr}[T_\mu^{I(0)}(k, l) w_{\mu'}^\mu \Phi^{\mu'}(k, l)] \quad (52)$$

where the parton amplitude $T_\mu^{I(0)}(k, l)$ is defined as

$$T_\mu^{I(0)}(k, l) = [(i\gamma_\mu) \frac{i(\not{y} + \not{k})}{(l+k)^2 + i\epsilon} + \frac{-i(\not{q} - \not{l} + \not{k})}{(q-l+k)^2 + i\epsilon} (-i\gamma_\mu)] (1 + \gamma_5) \quad (53)$$

and the meson amplitude $\Phi^{\mu'}(k, l)$

$$\Phi^{\mu'}(k, l) = \int d^4y \int d^4z e^{il \cdot z} e^{ik \cdot (y-z)} \langle M_2 | \bar{q}_2(z) (-g\not{A}^{\mu'}(y)) q_3(0) | 0 \rangle . \quad (54)$$

The tensor $w_{\mu'}^\mu = g_{\mu'}^\mu - q^\mu n_{\mu'}$ has been introduced. Note that, for convenience, we have made a change of variables for the loop parton momenta $l = l_1^\mu$ and $k^\mu = (l_2 - l_1)^\mu$. We assume

that the emitted M_2 meson is highly energetic. As shown in last section, the dominante configuration is composed of collinear l_1 and l_2 . This allows us to expand the parton amplitude $T_{ij,\alpha}^{I(0)}(k, l)$ with respect to $\hat{l} = xq$ and $\hat{k} = (x' - x)q$

$$T_{ij,\mu}^{I(0)}(k, l) = T_{\mu}^{I(0)}(\hat{k}, \hat{l}) + \left. \frac{\partial T_{ij,\mu}^{I(0)}(k, l)}{\partial l^\nu} \right|_{l=\hat{l}, k=\hat{k}} (l - \hat{l})^\nu + \left. \frac{\partial T_{ij,\mu}^{I(0)}(k, l)}{\partial k^\nu} \right|_{l=\hat{l}, k=\hat{k}} (k - \hat{k})^\nu + \dots \quad (55)$$

Substituting the first term back into the convolution integrations gives

$$\int dx \int dx' \text{Tr}[T_{ij,\mu}^{I(0)}(x, x') w_{\mu'}^\mu \Phi^{\mu'}(x, x')] \quad (56)$$

where

$$\begin{aligned} \Phi^{\mu'}(x', x) = & \int \frac{d^4 l}{(2\pi)^4} \delta(x - l \cdot n) \int \frac{d^4 k}{(2\pi)^4} \delta(x' - x - k \cdot n) \int d^4 y \int d^4 z \\ & \times e^{il \cdot z} e^{ik \cdot y} \langle M_2 | \bar{q}_2(z) (-g A^{\mu'}(y)) q_3(0) | 0 \rangle . \end{aligned} \quad (57)$$

In the above collinear limit step $T_{ij,\mu}^{I(0)}(l, k) \rightarrow T_{ij,\mu}^{I(0)}(x, x')$, there arises an infrared divergence as $x' \rightarrow 0$, which is from the denominators of virtual quark propagators

$$\frac{ix' \not{q}}{(x'q)^2 + i\epsilon} . \quad (58)$$

We regularize this divergence by the following method. Since the full quark propagator with momentum $l' = l + k$ can be decomposed into its long distance part and short distance part as

$$\frac{i \not{l}}{(l+k)^2 + i\epsilon} = \frac{i \not{l}_L}{(l')^2 + i\epsilon} + \frac{i \not{l}}{2n \cdot l'} . \quad (59)$$

The long distance part gives vanishing result upto twist-3. The short distance part is absorbed by the parton amplitude. The divergence is then regularized by replacing the quark propagators with its corresponding special propagators [7, 40]

$$\frac{ix' \not{q}}{(x'q)^2 + i\epsilon} \rightarrow \frac{i \not{x}}{2x + i\epsilon} \frac{x' - x}{x' - x + i\epsilon} . \quad (60)$$

The introduction of a special propagator for an on-shell fermion propagator is due to the fact that the fermion propagators in Fig. 3(a) and 3(b) become on-shell and divergent after the collinear expansion. The divergent part of these propagators leads to long distance contributions that should be included into the twist-2 distribution amplitude for the M_2 meson. However, there are also finite contact part of these propagators, which leads to

contributions of one twist higher. The more detailed explanation about the meaning of the special propagator refers to [7, 40].

Under light-cone gauge $n \cdot A(y) = 0$, it is convenient to transform the gluon fields $A^\mu(y)$ into its field strength $G^{\nu\mu}(y)$ by using following replacement

$$A^{\mu'}(y) \rightarrow \frac{in_\nu G^{\nu\mu'}(y)}{(x' - x)}, \quad (61)$$

and

$$\Phi_{M_2}^{\mu'}(x, x') \rightarrow \frac{in_\nu}{x' - x} \Phi_{M_2}^{\nu\mu'}(x, x'). \quad (62)$$

The factor $in_\nu/(x' - x)$ is then absorbed by $T_\mu^{I(0)}(x, x')$ into the form

$$T_{\mu\nu}^{I(0)}(x, x') \equiv T_\mu^{I(0)}(x, x') \frac{in_\nu}{x' - x}. \quad (63)$$

The factorization of the spin indices gives

$$\frac{1}{8} \int dx \int dx' \text{Tr}[T_{\mu\nu}^{I(0)}(x, x') \sigma_{\alpha\beta} \gamma_5] w_\mu^\nu \text{Tr}[\sigma^{\alpha\beta} \gamma_5 \Phi^{\nu\mu'}(x, x')] + \dots, \quad (64)$$

in which other spin decompositions give higher twist contributions.

The numerators in the contraction $\text{Tr}[T_{\mu\nu}^{I(0)} \sigma_{\alpha\beta} \gamma_5]$ can give terms proportional to $n_\nu n_\mu (q_\alpha n_\beta - n_\alpha q_\beta)$, $n_\nu d_{\perp, \mu\mu''} (q_\alpha n_\beta - n_\alpha q_\beta)$, and $n_\nu d_{\perp, \mu\mu''} \epsilon_{\perp, \alpha\beta}$. The transversal tensors $d_{\perp, \alpha\beta}$ and $\epsilon_{\perp, \alpha\beta}$ are defined as $d_{\perp, \alpha\beta} = q_\alpha n_\beta + q_\beta n_\alpha - g_{\alpha\beta}$ and $\epsilon_{\perp, \alpha\beta} = \epsilon_{\alpha\beta\eta\lambda} q^\eta n^\lambda$. The trace of $d_{\perp, \alpha\beta}$ is defined to be negative $d_{\perp, \alpha}^\alpha = -2$. Since ν and μ indices in $\Phi_{M_2}^{\nu\mu}$ are antisymmetric under $\mu \leftrightarrow \nu$, the terms proportional to $n_\nu n_\mu (q_\alpha n_\beta - n_\alpha q_\beta)$ then vanish. For those terms proportional to $n_\nu d_{\perp, \mu\mu''} (q_\alpha n_\beta - n_\alpha q_\beta)$, as they are contracted with $\text{Tr}[\sigma^{\alpha\beta} \gamma_5 \Phi^{\nu\mu'}(x, x')]$, the q_α factor in $n_\nu d_{\perp, \mu\mu''} (q_\alpha n_\beta - n_\alpha q_\beta)$ results in twist-4 contributions by using the property of the long distance propagator of the quark fields. The terms proportional to $n_\nu d_{\perp, \mu\mu''} \epsilon_{\perp, \alpha\beta}$ lead to twist-3 contributions. The final result appears as

$$\int dx \int dx' \frac{G_\mu^\beta(x, x') n^\mu n_\beta}{(x' - x)x}, \quad (65)$$

where the function $G_\mu^\beta(x, x')$ is defined as

$$\begin{aligned} G_\mu^\beta(x, x') = & \int \frac{d^4 l}{(2\pi)^4} \delta(x - l \cdot n) \int \frac{d^4 k}{(2\pi)^4} \delta(x' - x - k \cdot n) \int d^4 y \int d^4 z \\ & \times e^{il \cdot z} e^{ik \cdot y} \langle M_2 | \bar{q}_2(z) \sigma_{\mu\alpha} \gamma_5 w_{\alpha'}^\alpha g G^{\beta\alpha'}(y) q_3(0) | 0 \rangle. \end{aligned} \quad (66)$$

Note that we have used the G -parity symmetry $x \leftrightarrow \bar{x}'$ to simplify the above result. This assumption is valid for π mesons, but may not be appropriate for the K or η mesons. Therefore, it is noted that, in the above result, there exist symmetry breaking effects for K and η mesons. However, we will ignore such a corrections from the symmetry breaking in the following calculations. By referring to the definition [41]

$$\begin{aligned} & \langle M_2 | \bar{q}_2(z) \sigma_{\mu\nu} \gamma_5 g G_{\alpha\beta}(y) q_3(0) | 0 \rangle \\ &= -i \frac{f_{M_2} m_{M_2}^2}{m_{q_2} + m_{\bar{q}_3}} (q_\alpha q_\mu d_{\perp, \nu\beta} - q_\alpha q_\nu d_{\perp, \mu\beta} - q_\beta q_\mu d_{\perp, \nu\alpha} + q_\beta q_\nu d_{\perp, \alpha\mu}) T(z, y) + \dots, \end{aligned} \quad (67)$$

where

$$T(z, y) = \int_0^1 dx \int_0^{\bar{x}} dx' e^{-ixq \cdot z} e^{-i(x-x')q \cdot y} T(x, x'), \quad (68)$$

we can arrive at the result

$$\begin{aligned} & \int dx \int dx' \frac{G_\mu^\beta(x, x') n^\mu n_\beta}{(x' - x)x} \\ &= -\frac{2if_{M_2} m_{M_2}^2}{m_{q_2} + m_{\bar{q}_3}} \int dx \int dx' \frac{T(x, x')}{(x' - x)x}. \end{aligned} \quad (69)$$

By using the normalization for $\langle M_1 | \bar{q}_1(0) (1 - \gamma_5) b(0) | \bar{B} \rangle$, it is easy to derive the tree level three parton contributions for operator O_6 as

$$\langle O_6 \rangle_{1-gluon} = \frac{2A_{M_2}^{G3} m_{M_2}^2}{m_b(m_{q_2} + m_{\bar{q}_3})} \langle O_1 \rangle_f \quad (70)$$

with

$$A_{M_2}^{G3} = 2 \int_0^1 dx \int_0^{\bar{x}} dx' \frac{T_{M_2}(x', x)}{(x' - x)x}. \quad (71)$$

We now explain the expansion with the covariant gauge $\partial \cdot A = 0$. We first decompose $A^\mu(y)$ into its longitudinal and transversal components as $A^\mu(y) = n \cdot A(y) q^\mu + d_{\perp, \mu'}^\mu A^{\mu'}(y)$. The transversal part $d_{\perp, \mu'}^\mu A^{\mu'}(y)$ results in contributions of higher than twist-3. The longitudinal part $n \cdot A(y) q^\alpha$ gives twist-3 contributions. Similar to the light-cone gauge, we need to transform the gluon fields into its field strength. Here, it needs one transversal momentum k_\perp factor from expansion of the parton amplitude $T_\mu^{I(0)}(l, k)$ in Eq. (55). The contraction of $T_\mu^{I(0)}(x, x')$ with $q^\mu n \cdot \Phi_{M_2}(x, x')$ leads to two parton gauge phase factor

$$\text{Tr}[T_\mu^{I(0)}(x, x') q^\mu n \cdot \Phi_{M_2}(x, x')] = \text{Tr}\left[\frac{T^{I(0)}(x') - T^{I(0)}(x)}{x' - x} n \cdot \Phi_{M_2}(x, x')\right] \quad (72)$$

It is convenient to write $(k - \hat{k})^\rho = d_{\perp\rho'}^\rho(k - \hat{k})^{\rho'} + q \cdot k n^\rho$. Only transversal part $k_\perp^\rho = d_{\perp\rho'}^\rho(k - \hat{k})^{\rho'}$ contributes at twist-3. This is because the term $\partial T_\mu^{I(0)}/\partial k^\nu$ can have terms proportional to $g_{\mu\nu}$ and $\sigma_{\mu\nu}$. The terms related to $q \cdot k n^\nu$ leads to twist-4 contributions.

For the transversal part k_\perp , only $\sigma_{\mu\nu}$ terms can contribute. Let the k_\perp^ρ factor absorbed into $\Phi^\mu(l, k)$ and use the replacement $k_\perp^\nu A^\mu(y) \rightarrow -iG^{\nu\mu}(y)$, we can derive the result

$$\begin{aligned} \langle O_6 \rangle_{1-gluon}^{t=3} = & -2 \int dx \int dx' \text{Tr} \left[\frac{\partial T_\mu^{I(0)}(x, x')}{\partial k^\nu} G^{\nu\mu}(x, x') \right] \\ & \times \langle M_1 | \bar{q}_1(0) (1 - \gamma_5) b(0) | \bar{B} \rangle, \end{aligned} \quad (73)$$

where

$$\frac{\partial T_\mu^{I(0)}(x', x)}{\partial k^\nu} = \frac{-i\sigma_{\mu\nu}}{(x' - x)xq^2} (1 + \gamma_5) \quad (74)$$

and

$$\begin{aligned} G^{\nu\mu}(x, x') = & \int \frac{d^4 l}{(2\pi)^4} \delta(x - l \cdot n) \int \frac{d^4 k}{(2\pi)^4} \delta(x' - x - k \cdot n) \int d^4 y \int d^4 z \\ & \times e^{il \cdot z} e^{ik \cdot y} \langle M_2 | \bar{q}_2(z) i g G^{\nu\mu}(y) q_3(0) | 0 \rangle. \end{aligned} \quad (75)$$

The contraction of $\sigma_{\mu\nu}$ with $G^{\nu\mu}(x, x')$ gives

$$\text{Tr}[i\sigma_{\mu\nu} G^{\nu\mu}(x, x')] = \frac{-2if_{M_2} m_{M_2}^2 q^2}{(m_{q_2} + m_{\bar{q}_3})} T(x, x'). \quad (76)$$

Note that the q^2 factor in the denominator of $\partial T_\mu^{I(0)}(x', x)/\partial k^\nu$ is cancelled by the q^2 factor in the numerator of $\text{Tr}[i\sigma_{\nu\mu} G^{\nu\mu}(x, x')]$. It is easy to see that Eq. (73) is equal to the result derived from the light-cone gauge. This explicitly shows the gauge invariance of the three parton contributions.

There are related diagrams, such as those in Fig. 3(c) and 3(d). Because the spectator quark of the \bar{B} meson can carry only a soft momentum, this makes the relevant contributions associated with Fig. 3(c) and 3(d) dominated by soft gluons as the form factors $F_{+,0}^{B \rightarrow M_1}$. In addition, the relevant contributions are of $O(m_b^{-2})$ with respect to the leading twist amplitude. It can be understood as following. The sum of the lower parts of the diagrams in Fig. 3(c) and 3(d) is proportional to

$$\langle M_1 | \bar{q}_1 \left[\frac{2p_\nu + \gamma_\nu \not{k}}{2p \cdot k} \Gamma_i - \Gamma_i \frac{2P_{b\nu} - \not{k} \gamma_\nu}{2P_b \cdot k} \right] b | \bar{B} \rangle, \quad (77)$$

where k is the momentum of the gluon from M_2 and the equation of motions for b and q_1 quarks have been used. After taking the collinear limit, $k \rightarrow x'q$, we write the expression as

$$Aq_\nu + B_{\mu\nu}q^\mu, \quad (78)$$

where

$$A \equiv \frac{1}{x'Q^2} \langle M_1 | \bar{q}_1 \Gamma_i | \bar{B} \rangle, \quad (79)$$

$$B_{\mu\nu} \equiv \frac{1}{2x'Q^2} \langle M_1 | \bar{q}_1 (\gamma_\nu \gamma_\mu \Gamma_i + \Gamma_i \gamma_\mu \gamma_\nu) b | \bar{B} \rangle. \quad (80)$$

For light-cone gauge, only A term contributes. As for the covariant gauge, only $B_{\mu\nu}$ term contributes. The only contributions come from $(V-A)(V \pm A)$ operators. This implies that the upper parts of the diagrams in Fig. 3(c) and 3(d) are proportional to the twist-4 LCDA of M_2 . The combination of the upper and the lower parts gives a $O(m_b^{-2})$ contributions with respect to the leading twist amplitude.

There are possibilities that the additional gluon of the M_2 meson can interact with the spectator quark of the \bar{B} meson. Since the spectator quark carries a soft momentum, the momentum conservation at the interaction vertex prevents the momentum of the gluon from being collinear to the M_2 meson's momentum. Therefore, there require additional radiative gluons interacting between the other parton lines and the spectator quark line to make the momentum of the gluon to be collinear to the M_2 meson's momentum. This results in contributions of order $O(\alpha_s)$. We identify the relevant contributions as $O(\alpha_s)$ three parton corrections. As mentioned previously, we plan to discuss these contributions in other places [44].

The total twist-3 contribution from operator O_6 is then equal to

$$\langle O_6 \rangle^{t=3} = \frac{2(1 + A_{M_2}^{G3})m_{M_2}^2}{m_b(m_{q_2} + m_{\bar{q}_3})} \langle O_1 \rangle_f. \quad (81)$$

For operator O_8 , there are similar results.

IV. APPLICATIONS

For penguin dominant $B \rightarrow \pi K$ decays, the relevant decay amplitudes under QCD factorization are parametrized as the following [3]

$$\begin{aligned}
A(B^- \rightarrow \pi^- \bar{K}^0) &= \lambda_p \left[(a_4^p - \frac{1}{2}a_{10}^p) + r_\chi^K (a_6^p - \frac{1}{2}a_8^p) \right] A_{\pi K} \\
&\quad + (\lambda_u b_2 + (\lambda_u + \lambda_c)(b_3 + b_3^{EW})) B_{\pi K} , \\
-\sqrt{2}A(B^- \rightarrow \pi^0 K^-) &= [\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p) + \lambda_p r_\chi^K (a_6^p + a_8^p)] A_{\pi K} \\
&\quad + [\lambda_u a_2 + \lambda_p \frac{3}{2}(-a_7 + a_9)] A_{K\pi} \\
&\quad + (\lambda_u b_2 + (\lambda_u \lambda_c)(b_3 + b_3^{EW})) B_{\pi K} , \\
-A(\bar{B}^0 \rightarrow \pi^+ K^-) &= [\lambda_u a_1 + \lambda_p(a_4^p + a_{10}^p) + \lambda_p r_\chi^K (a_6^p + a_8^p)] A_{\pi K} \\
&\quad + ((\lambda_u + \lambda_c)(b_3 - \frac{1}{2}b_3^{EW})) B_{\pi K} \\
\sqrt{2}A(\bar{B}^0 \rightarrow \pi \bar{K}^0) &= A(B^- \rightarrow \pi^- \bar{K}^0) + \sqrt{2}A(B^- \rightarrow \pi^0 K^-) - A(\bar{B}^0 \rightarrow \pi^+ K^-) \quad (82)
\end{aligned}$$

where $\lambda_p = V_{pb}V_{ps}^*$, $a_i \equiv a_i(\pi K)$, and $\lambda_p a_i^p = \lambda_u a_i^u + \lambda_c a_i^c$. The CP conjugation of decay amplitudes are obtained by replacing $\lambda_p \rightarrow \lambda_p^*$ for the above amplitudes. The factorized matrix elements are defined as

$$\begin{aligned}
A_{\pi K} &= i \frac{G_F}{\sqrt{2}} (m_B^2 - m_\pi^2) F_0^{B \rightarrow \pi}(m_K^2) f_K , \\
A_{K\pi} &= i \frac{G_F}{\sqrt{2}} (m_B^2 - m_K^2) F_0^{B \rightarrow K}(m_\pi^2) f_\pi . \quad (83)
\end{aligned}$$

The form factors are defined

$$\langle P(p) | \bar{q} \gamma^\mu b | \bar{B} \rangle = F_+^{B \rightarrow P}(q^2) (P_B^\mu + p^\mu) + [F_0^{B \rightarrow P}(q^2) - F_+^{B \rightarrow P}(q^2)] \frac{m_B^2 - m_P^2}{q^2} q^\mu . \quad (84)$$

The form factors coincide as $q^2 = 0$, $F_+^{B \rightarrow P}(0) = F_0^{B \rightarrow P}(0)$. The expressions for the parameters a_i are referred to [2, 3]. For numerical calculations, we will use the following input parameters

$$\begin{aligned}
\Lambda_{MS}^{(5)} &= 0.225 \text{GeV} , \quad m_b(m_b) = 4.2 \text{GeV} , \quad m_c(m_b) = 1.3 \text{GeV} , \quad m_s(2 \text{GeV}) = 0.090 \text{GeV} , \\
|V_{cb}| &= 0.41 , \quad |V_{ub}/V_{cb}| = 0.09 , \quad \gamma = 70^\circ , \quad \tau(B^-) = 1.67 \text{ps} , \\
\tau(B_d) &= 1.54 \text{ps} , \quad f_\pi = 131 \text{MeV} , \quad f_K = 160 \text{MeV} , \quad f_B = 200 \text{MeV} , \\
F_0^{B \rightarrow \pi} &= 0.28 , \quad F_0^{B \rightarrow K} = 0.34 . \quad (85)
\end{aligned}$$

For λ_u and λ_c , we take the following convention for their parametrization

$$\frac{\lambda_u}{\lambda_c} = \tan^2 \theta_c R_b e^{-i\gamma} \quad (86)$$

where

$$\begin{aligned}\tan^2 \theta_c &= \frac{\lambda^2}{1 - \lambda^2} , \\ R_b &= \frac{1 - \lambda^2/2}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| , \\ \lambda &= |V_{us}| .\end{aligned}\tag{87}$$

The value of λ is taken as 0.22.

By using previous input parameters, we list the values of a_i , $i = 1, \dots, 10$, and b_j , $j = 1, \dots, 3$, calculated at the scale $m_b = 4.2\text{GeV}$ as below

$$\begin{aligned}a_1 &= 0.995 + 0.018i , & a_2 &= 0.209 - 0.104i , & a_3 &= -0.003 + 0.003i , \\ a_4^u &= -0.031 - 0.013i & a_4^c &= -0.030 + 0.027i , & a_5 &= 0.007 - 0.004i , \\ r_\chi^K a_6^u &= -0.050 - 0.015i , & r_\chi^K a_6^c &= -0.047 - 0.005i , & a_7/\alpha &= 0.007 + 0.006i , \\ r_\chi^K a_8^u/\alpha &= 0.087 - 0.043i , & r_\chi^K a_8^c/\alpha &= 0.094 - 0.021i , & a_9/\alpha &= -1.135 - 0.024i , \\ a_{10}^u/\alpha &= -0.175 + 0.093i , & a_{10}^c/\alpha &= -0.175 + 0.093i , & r_A b_1 &= 0.021 , \\ r_A b_2 &= -0.008 , & r_A b_3 &= -0.006 , & r_A b_3^{EW}/\alpha &= -0.018 ,\end{aligned}\tag{88}$$

in which

$$r_A = \frac{B_{\pi K}}{A_{\pi K}} = \frac{f_B f_\pi}{m_B^2 F_0^{B \rightarrow \pi}(0)} ,\tag{89}$$

and

$$r_\chi^K = \frac{2m_K^2}{m_b(m_q + m_s)} .\tag{90}$$

The tree level three parton contributions modify the parameters r_χ^K as $r_\chi^K(1 + A_{M_2}^{G_3})$ with parameter $A_{M_2}^{G_3}$ defined in Eq. (71). The value of $A_{M_2}^{G_3}$ depends on the model of the three parton distribution amplitude $T_{M_2}(x, x')$. Here we employ the model derived from the light-cone sum rule [41]

$$T(x, x') = 360\eta x x' (x - x')^2 (1 + \frac{\omega}{2}(7(x - x') - 3)),\tag{91}$$

where the parameters are assumed to be $\eta = 0.015$ and $\omega = -3.0$ for $M_2 = \pi, K$, or η . This give us $A_{M_2}^{G_3} = 0.585$.

The branching ratio for a $\bar{B} \rightarrow \pi K$ decay is given by this expressions

$$Br(\bar{B} \rightarrow \pi K) = \frac{\tau_B}{16\pi m_B} |A(\bar{B} \rightarrow \pi K)|^2 .\tag{92}$$

We can use the above formula to predict CP averaged branching ratios for $B \rightarrow \pi K$ decays. The predictions with three parton corrections in units of 10^{-6} are given as

$$\begin{aligned}
Br(B^- \rightarrow \pi^- \bar{K}^0) &= 19.0 , \\
Br(B^- \rightarrow \pi^0 K^-) &= 10.0 , \\
Br(\bar{B}^0 \rightarrow \pi^+ K^-) &= 16.1 , \\
Br(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 7.7 .
\end{aligned} \tag{93}$$

For comparison, we also list the predictions with only two parton contributions in units of 10^{-6} in the following,

$$\begin{aligned}
Br(B^- \rightarrow \pi^- \bar{K}^0) &= 11.2 , \\
Br(B^- \rightarrow \pi^0 K^-) &= 6.1 , \\
Br(\bar{B}^0 \rightarrow \pi^+ K^-) &= 9.4 , \\
Br(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 4.4 .
\end{aligned} \tag{94}$$

For reference, we enlist the experimental data in units of 10^{-6} summarized by the HFAG group [42]

$$\begin{aligned}
Br(B^- \rightarrow \pi^- \bar{K}^0) &= 23.1 \pm 1.0 , \\
Br(B^- \rightarrow \pi^0 K^-) &= 12.8 \pm 0.6 \\
Br(\bar{B}^0 \rightarrow \pi^+ K^-) &= 19.7 \pm 0.6 \\
Br(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 10.0 \pm 0.6 .
\end{aligned} \tag{95}$$

By comparing the predictions with or without the three parton corrections, one may notice that the predicted branching ratios are significantly enhanced by about $1.65 \sim 1.75$ times in their magnitudes.

The two parton predictions for the $\bar{B} \rightarrow \pi K$ decays made here are consistent with the findings of previous studies using QCD factorization approach [3, 43]. The two parton predictions made in [43] are much lower than the experimental data under the QCD factorization approach. Because the calculations of the three parton corrections were inaccessible in their studies, this led them to conclude that the QCD factorization is impossible to explain the penguin dominant $\bar{B} \rightarrow \pi K$ decays. The two parton predictions made in [3] for $\bar{B} \rightarrow \pi K$ decays are still lower than the data. Only extending the predictions by using

extreme limits of input parameters can make the predictions to be consistent with the measurements. This seems not a reasonable solution from the theoretical point of view. On the other hand, as shown in the above, our approach has shown that the predictions with three parton contributions are more close to the data than the two parton predictions. Although the predictions with three parton contributions are still lower than the experimental data, the $O(\alpha_s)$ corrections can improve the predictions.

V. DISCUSSIONS AND CONCLUSIONS

The significance of the three parton contributions for penguin dominant $\bar{B} \rightarrow \pi K$ decays can also be seen from a phenomenological point of view. By appropriate arrangement, the parameters a_i can be calculated for $B \rightarrow \pi\pi$ decays. For the pure penguin $B^- \rightarrow \pi^- \bar{K}^0$, its dominant contributions arise from the $|a_4^c(\pi K) + r_\chi^K a_6^c(\pi K)|$ term. The uncertainty due to the form factors can be eliminated by considering the ratio between the decay rates of $B^- \rightarrow \pi^- \bar{K}^0$ and $B^- \rightarrow \pi^- \pi^0$ as the following

$$\begin{aligned} \left| \frac{a_4^c(\pi K) + r_\chi^K a_6^c(\pi K)}{a_1(\pi\pi) + a_2(\pi\pi)} \right| &= \frac{|V_{ub}|}{|V_{cb}|} \frac{f_\pi}{f_K} \left[\frac{\Gamma(B^- \rightarrow \pi^- \bar{K}^0)}{2\Gamma(B^- \rightarrow \pi^- \pi^0)} \right]^{1/2} \\ &= 0.105 \pm 0.001, \end{aligned} \quad (96)$$

where the error comes from the branching ratios. In the above, we have used the branching ratio $Br(B^- \rightarrow \pi^- \pi^0) = 5.7 \pm 0.4$. According to QCD factorization calculations, $|a_1(\pi\pi) + a_2(\pi\pi)| = 1.17$ and $|a_4^c(\pi K) + r_\chi^K a_6^c(\pi K)| = 0.08$ with only two parton contributions. This gives the prediction of the ratio to be 0.066, which is lower than the experimental value. By adding the tree level three parton contributions, the factor r_χ^K then becomes $r_\chi^K(1 + A_K^{G3})$ and makes $|a_4^c(\pi K) + r_\chi^K(1 + A_K^{G3})a_6^c(\pi K)| = 0.104$. The predicted ratio is changed to be 0.089, which is closer to the two parton prediction. The above fact may also indicate that the three parton corrections could be important for understanding the penguin dominant $\bar{B} \rightarrow \pi K$ decays under the QCD factorization approach.

In order to make sure that the three parton contributions are indeed significant and also compatible with the QCD factorization, an important task is to finish $O(\alpha_s)$ calculations for the three parton contributions. The related work in this direction has been proceeded and will be reported in our another preparing paper [44].

There are similar three parton contributions having been calculated under the light-cone sum rule [45, 46]. In [45], the twist-3 three parton contributions associated with soft gluons are calculated in the framework of light-cone sum rule. The contributions are shown negligible in $\bar{B} \rightarrow \pi\pi$ decays. In [46], the contributions from the diagrams similar to Fig. 3(c) and 3(d) were calculated for $B \rightarrow \pi\omega$ decays. They are found to be twist-4 and vanishing. In addition, significant effects were found due to the three parton Fock state of the π in the $B \rightarrow \pi\omega$ decays. Since they are dominated by soft gluons, it is better determined by QCD sum rule. As shown in [46], in the Euclidean region of $(p+q)^2$, the relevant contributions are from the twist-3 and twist-4 three parton LCDAs of the π . As mentioned before, we identify these power corrections as non-partonic ones. From the theoretical point of view, we suggest that the partonic and non-partonic power corrections should be distinguished under the QCD factorization, although they may be equally important in phenomenology.

For comparison, we employ the replacing rules for the a_i coefficients [46] to account for the three parton effects from the M_1 meson. The rule is

$$\begin{aligned} a_{2i} &\rightarrow a_{2i} + [1 + (-1)^{\delta_{3i}+\delta_{4i}}]C_{2i-1}f_3/2, \\ a_{2i-1} &\rightarrow a_{2i-1} + (-1)^{\delta_{3i}+\delta_{4i}}C_{2i}f_3, \end{aligned} \quad (97)$$

where $i = 1, \dots, 5$, and C_i are the Wilson coefficients calculated at the scale $\mu_h = 1.45$ GeV, and $f_3 = 0.12$, which is assumed to be universal. With these three parton corrections, the predicted CP averaged branching ratios for $\bar{B} \rightarrow \pi K$ in units of 10^{-6} are

$$\begin{aligned} Br(B^- \rightarrow \pi^- \bar{K}^0) &= 9.5, \\ Br(B^- \rightarrow \pi^0 K^-) &= 5.9, \\ Br(\bar{B}^0 \rightarrow \pi^+ K^-) &= 8.4, \\ Br(\bar{B}^0 \rightarrow \pi^0 \bar{K}^0) &= 3.4, \end{aligned} \quad (98)$$

which becomes smaller than those predictions in Eq. (94).

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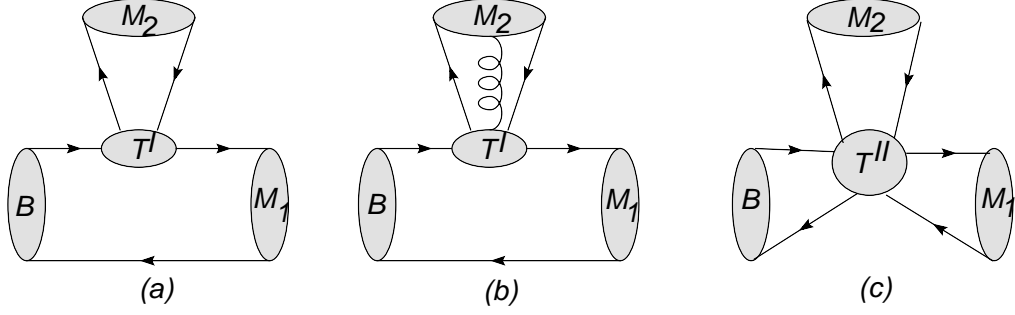


FIG. 1: The parton topologies correspond to the parton amplitudes of four, five and six parton interactions, respectively.

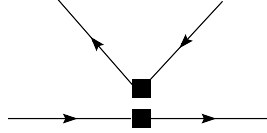


FIG. 2: The Feynman diagram describes the tree level four parton amplitude, $T_{ij}^{I(0)}$. The square symbol represents the vertex of weak interactions.

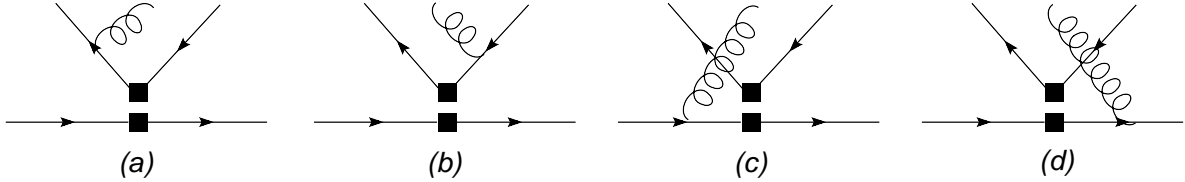


FIG. 3: The Feynman diagrams describe the tree level five parton amplitude, $T_{ij,\mu}^{I(0)}$. The square symbol represents the vertex of weak interactions.

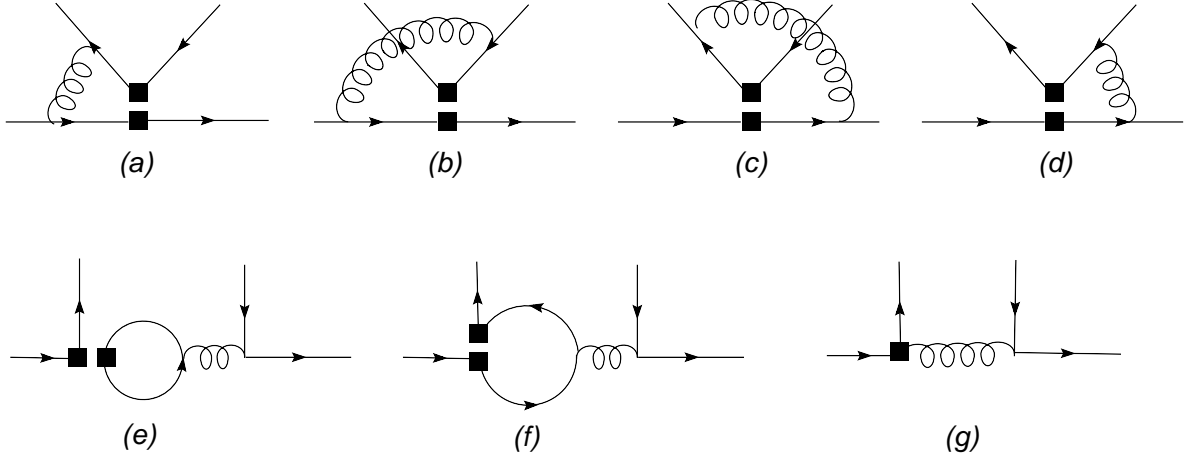


FIG. 4: The Feynman diagrams describe the $O(\alpha_s)$ four parton amplitude, $T_{ij}^{I(1)}$. The square symbol represents the vertex of weak interactions.

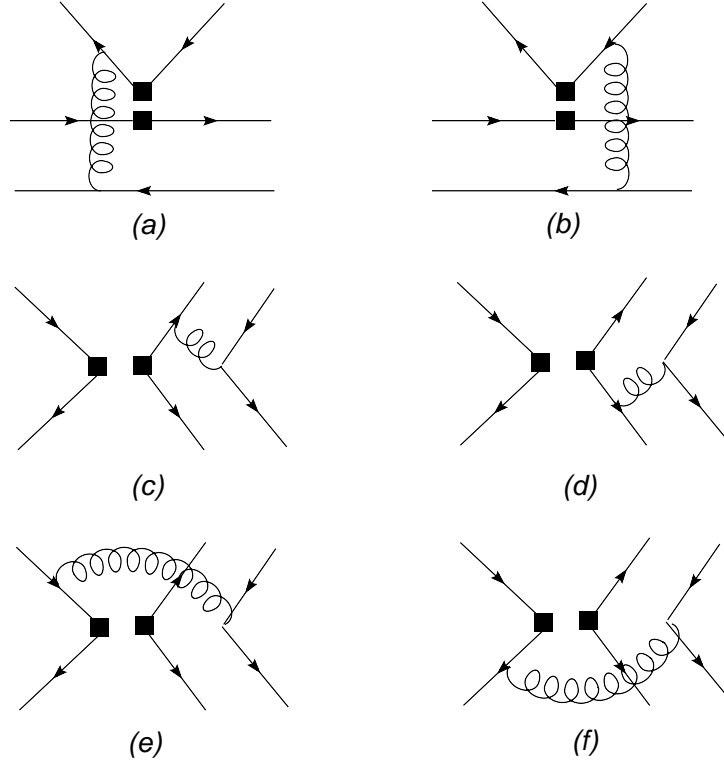


FIG. 5: The Feynman diagrams for the $O(\alpha_s)$ six parton amplitude, $T^{II(1)}$. The square symbol represents the vertex of weak interactions.

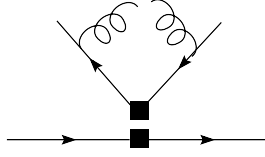


FIG. 6: The Feynman diagram for the six parton amplitude with $|q\bar{q}gg\rangle$ Fock state. The square symbol represents the vertex of weak interactions.